Optimal Fiscal and Monetary Policy: Equivalence Results*

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Abstract

In this paper we analyze the way in which restrictions on price setting behavior shapes optimal fiscal and monetary policies in dynamic general equilibrium monetary models. We first show that the set of allocations that can be implemented as equilibria with taxes is independent of the price setting restrictions. We then derive two simple rules for optimal policy that are independent of the price setting restrictions. First, fiscal policy should be chosen as if all prices were flexible. Second, monetary policy must replicate the flexible prices allocation, as if all prices were sticky.

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1 Introduction

In this paper we analyze the implications of restrictions on price setting behavior for the conduct of fiscal and monetary policy, in dynamic general equilibrium monetary models. First, we analyze the impact of these price setting restrictions on the set of allocations that can be implemented as a general equilibrium with taxes. Then, we study, given any implementable allocation, how the policies that implement it depend on the price setting restrictions.

The main contribution of this paper is to study fiscal and monetary policy in environments with sticky prices following the tradition of general equilibrium dynamic Ramsey problems. Thus, as in Ireland (1996), Carlstrom and Fuerst (1998), King and Wolman (1998), Khan, King and Wolman (2000) and Adao, Correia and Teles (2000), the relationship between policies and allocations is explicitly derived from a general equilibrium model. In contrast to that literature, however, our approach allows us to jointly study monetary and fiscal policy. In particular, we consider scenarios in which first best outcomes cannot be implemented.

The optimal stabilization policy discussion has always been centered around the impact of price stickiness on the mapping between policies and allocations. Thus, the non-neutrality of money in the presence of price stickiness has provided monetary policy with a protagonic role in this discussion. Ours, is the first attempt at studying optimal stabilization policy in a general equilibrium dynamic model in which both monetary and fiscal instruments are considered in discussing the mapping from policies to allocations.

We compare flexible prices economies with economies in which prices are set one period in advance, but are identical in all other respects. We also consider mixed environments, were there are both flexible and sticky prices firms. The main finding of the paper is that the set of allocations that a government can implement is independent of the price setting rule. In particular, if as it is standard in Ramsey problems we assume a benevolent government, the second best allocation is the same, regardless of the price setting rules. On one hand, this result suggests that sticky prices are redundant. On the other, they show that the characterization of the optimal distortions is independent of the restrictions on price setting. Therefore, the existing results

\footnote{In Correia, Nicolini and Teles (2001b) we show these results extend to small open economies.}

We also characterize the policies that implement the Ramsey allocation. In the economy with flexible prices, where money is neutral - but not superneutral-, the fiscal instruments and the nominal interest rate determine the allocations. In these economies the optimal money supply is indeterminate, since it only affects the current price level. Under sticky prices, where money is not neutral, we obtain analogous results. In these environments, it is the state contingent fiscal policy that is indeterminate, suggesting short run neutrality of fiscal policy, or superneutrality of money. Finally, in environments where both firms that set prices in advance and firms that set prices contemporaneously coexist, both fiscal and monetary instruments are non-neutral such that both monetary and fiscal optimal policies are uniquely determined. It also turns out, that the indeterminacy created by neutrality results in the two extreme cases do not make the choices of optimal policies difficult, since the unique optimal policy in the mixed economies is also optimal in the two extremes.

Therefore, two simple principles for the conduct of optimal policy can be derived. In all these economies, fiscal policy must be chosen as if all prices were flexible, and monetary policy must replicate the flexible prices allocation, as if all prices were sticky. In summary, the optimal allocation is independent of the shares of sticky or flexible firms and therefore, results derived in Adao, Correia and Teles (1999) in a somewhat different environment apply: optimal policy is also independent of those shares.

The analysis of the paper is one very related to the discussion of the minimum set of policy instruments required to achieve a certain outcome. In the flexible prices economy, money is neutral, so the quantity of money has no power to determine allocations - although the rate of growth does. On the contrary, when prices are set-in-advance, the quantity of money can affect

\footnote{In a companion paper, Correia, Nicolini and Teles (2001c) we obtain that the Friedman rule is still optimal in these environments, and extend results in Zhu (1992) on optimal smoothing of taxes. Furthermore we consider restrictions on tax instruments both in flexible and sticky prices economies, and consider the implications for fiscal and monetary stabilization policy.}

\footnote{Adao, Correia and Teles (1999) also show that the policies that replicate flexible prices is independent of degree of not only price, but also portfolio stickiness.}
the allocations\textsuperscript{4}. The analysis of the paper shows that this additional policy instrument is redundant in standard general equilibrium dynamic Ramsey models, in the sense that it can affect allocations only in the way that standard fiscal policy also does.

The model we analyze introduces sticky prices in a very simple fashion. As most of the modern literature, we impose ad-hoc restrictions on the price setting process, instead of modelling the price setting decision and deriving the optimal price setting rules. We take as a fundamental parameter the fraction of firms that can adjust prices within a period. As such, the model is subject to the Lucas critique, and this raises doubts of its usefulness for policy analysis. We do not view this as a significant problem, since we show that both the optimal allocation and the optimal policy are independent of that assumed fundamental parameter. Thus, potential changes in the parameter due to changes in policy will not alter our conclusions regarding optimal policy.

The paper proceeds as follows. Section 2 presents the model and solves for equilibrium conditions on both the flexible and set-in-advance (SIA) prices economies. The main equivalence results are derived and a characterization of optimal policies is offered. Section 3 presents a model that combines firms that set prices in advance with firms without price setting restrictions and discusses properties of optimal policies. Section 4 concludes.

2 The model

Our model economy follows closely the structures in Ireland (1996), Carlstrom and Fuerst (1998) and Adao, Correia and Teles (2000). The state of the economy is represented by the realization of a random variable $\sigma_t \in \Sigma$. We let $\sigma^t = \{\sigma_0, \sigma_1, \sigma_2, ... \sigma_t\}$ denote a particular sequence of realizations of this random variable. Government expenditure shocks, $g_t = g(\sigma^t)$, and productivity shocks, $s_t = s(\sigma^t)$ are functions of the state. In addition, we let policy instruments to be functions of the state. That is, labor income taxes $\tau^h_t = \tau^h(\sigma^t)$, dividend taxes $\tau^d_t = \tau^d(\sigma^t)$, consumption taxes $\tau^c_t = \tau^c(\sigma^t)$ and money supplies $M_t = M(\sigma^t)$. These are all the natural policy instruments to consider in this environment.

As the effects of anticipated or unanticipated monetary policy are different, it is convenient to introduce the following notation. Let

\textsuperscript{4}See Adao, Correia and Teles (2000).
\[ M_t(\sigma^t) = M_{t-1}(\sigma^{t-1})\mu(\sigma^{t-1})\delta(\sigma^t), \]
\[ \overline{M}_t = E_{t-1}[M_t] = M_{t-1}(\sigma^{t-1})\mu(\sigma^{t-1}) \]

which implies

\[ E_{t-1}[\delta(\sigma^t)] = 1 \]

Thus, we identify the \( t-1 \) expected rate of money change between time \( t-1 \) and \( t \) with \( \mu(\sigma^{t-1}) \), while \( \delta(\sigma^t) \) represents the state contingent deviation on the rate of money growth.

The economy consists of a representative household, a continuum of producers of final goods indexed by \( i \in [0,1] \), and a government. Each firm produces a distinct, perishable consumption good, indexed by \( i \).

### 2.1 The households

Preferences are described by the expected utility function:

\[ U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - N_t) \right\} \]

where \( N_t \) is labor effort, \( \beta \in (0,1) \) is a discount factor and the composite \( C_t \) is

\[ C_t = \left[ \int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} \, di \right]^{\frac{\theta}{\theta-1}}, \theta > 1. \]

The households start period \( t \) with outstanding nominal wealth, \( W_t \), and decide to buy money balances, \( M_t \) and \( B_{t+1}^h \) units of nominal bonds that pay \( R_{t+1}B_{t+1}^h \) units of money one period later. They also buy \( Z_{t+1}^h \) units of state contingent nominal securities, that cost \( z_{t+1} \) in units of currency today, and each of them pays one unit of money at the beginning of period \( t+1 \) in a particular state. They can also buy \( A(i)_{t+1} \) units of stocks of firm \( i \), that cost \( a(i)_t \) in units of currency. Households have to pay labor income, dividend and consumption taxes.

The purchases of consumption goods have to be made with cash, so,
where \( P_t(i) \) is the money price of final good \( i \).

At the end of the period, households receive labor income, \( W_t N_t \) where \( W_t \) is the nominal wage rate, and collect dividends, given by current period profits \( d(i)_t \) that can be used to purchase consumption in the following period. Therefore, the households face the budget constraints

\[
M_t + B_{t+1}^h + E_t Z_{t+1}^h + \int_0^1 A(i)_{t+1}a(i)_t di \leq W_t
\]

\[
W_{t+1} = M_t + R_{t+1} B_{t+1}^h + Z_{t+1}^h - (1 + \tau_t^c) \int_0^1 P_t(i) c_t(i) di + \\
\int_0^1 A(i)_{t+1}a(i)_{t+1} di + W_t N_t (1 - \tau_n^p) + \int_0^1 A(i)_{t+1}d(i)_{t} di (1 - \tau_d^d)
\]

The problem of the consumer can be stated as maximizing \((1)\) subject to the cash in advance constraint, \((2)\), and the budget constraints \((3)\).

Let \( P_t = \left[ \int P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \). Then, the following marginal conditions summarize households behavior

\[
\frac{c_t(i)}{C_t} = \left( \frac{P_t(i)}{P_t} \right)^{-\theta}
\]

\[
\frac{u_{1-N_t}}{u_{Ct}} = \frac{W_t}{R_{t+1}} \frac{(1 - \tau_n^p)}{(1 + \tau_t^c)}
\]

\[
\frac{u_{Ct}}{P_t(1 + \tau_t^c)} = R_{t+1} E_t \left[ \frac{\beta u_{Ct+1}}{P_{t+1}(1 + \tau_{t+1}^c)} \right]
\]

\[
\frac{z_{t+1}}{P_t} = \frac{\beta u_{Ct+1}}{u_{Ct}} \frac{(1 + \tau_t^c)}{(1 + \tau_{t+1}^c)}
\]

\[
E_t \left[ z_{t+1} \right] = \frac{1}{R_{t+1}}
\]

\[
a(i)^{\frac{u_{Ct}}{P_t(1 + \tau_t^c)}} = E_t \left[ a(i)_{t+1} + d(i)_t (1 - \tau_d^d) \right] \frac{\beta u_{Ct+1}}{(1 + \tau_{t+1}^c) P_{t+1}}
\]
Condition (4) defines the demand for each of the intermediate goods \( i \) and condition (5) sets the intra-temporal marginal rate of substitution between consumption and leisure equal to the real wage times the corresponding taxes. Condition (6) is a requirement for the optimal savings decision. The last three conditions are arbitrage conditions for asset prices.

2.2 The government

The government must finance a given path of purchases \( \{G_t\}_{t=0}^{\infty} \), such that

\[
G_t = \left[ \int_0^1 g_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{1}{1-\theta}}, \theta > 1
\]

We assume that \( G_t > 0 \) for all \( t \) and all states. Given the prices on each good, \( P_t(i) \), the government minimizes the expenditure on government purchases by deciding according to

\[
\frac{g_t(i)}{G_t} = \left( \frac{P_t(i)}{P_t} \right)^{-\theta}
\]

A government policy consists of a sequence of a monetary and tax policy \( \{M_t, \tau^n_t, \tau^c_t, \tau^d_t\}_{t=0}^{\infty} \).

2.3 Firms

There is a continuum of competitively monopolistic firms, each one produces a single differentiated consumption good. The technology is linear in labor, the only production input.

The pricing equation for stocks (9) implies that the problem of maximizing the value of a single monopolistic firm can be written as the following dynamic programming problem

\[
a(i)_t \frac{u_{Cl}}{P_t(1 + \tau^c_t)} = \max_{\mu(i)} d(i)_t (1 - \tau^d_t) E_t \frac{\beta u_{Cl+1}}{(1 + \tau^c_{t+1})P_{t+1}} + E_t \left[ \frac{a(i)_{t+1} \beta u_{Cl+1}}{(1 + \tau^c_{t+1})P_{t+1}} \right]
\]

2.3.1 Flexible prices

Let \( \Xi_t = (1 - \tau^d_t) E_t \frac{\beta u_{Cl+1}}{(1 + \tau^c_{t+1})P_{t+1}} \). Note that \( \Xi_t \) does not depend on firm’s actions. Thus, as long as \( \Xi_t \geq 0 \), the solution, as of time \( t - 1 \), will be to set a state contingent rule for prices, satisfying
max \left\{ p(i, t) y_t (i) - W_t n_t (i) \right\}

subject to the demand function

\[
\frac{y_t (i)}{Y_t} = \left( \frac{P_t (i)}{P_t} \right)^{-\theta}
\]

obtained from the households problem (4), were \( Y_t = C_t + G_t \), and the technology

\[
y_t (i) \leq s_t n_t (i)
\]

where \( s_t \) is the level of technology. The optimal pricing rule is therefore

\[
P_t (i) \left[ 1 + \frac{d \ln P_t (i)}{d \ln y_t (i)} \right] - \frac{W_t}{s_t} = 0
\]

where \( \frac{d \ln P_t (i)}{d \ln y_t (i)} = -\frac{1}{\theta} \), so that \( \theta \) is the demand elasticity. Thus,

\[
P_t = P_t (i) = \frac{\theta W_t}{\theta - 1 s_t}
\]

The firms set a common price, a constant mark-up over their common marginal cost. Thus, given that technologies and demand functions are identical, an equilibrium will be characterized by equal prices and labor inputs across varieties. Therefore, we will drop indexes such that \( c_t (i) = C_t, n_t (i) = N_t \). In equilibrium,

\[
s_t = \frac{\theta W_t}{\theta - 1 P_t}
\]

Thus, as long as \( \Xi_t \geq 0 \), the rule \( P_t (i) = \frac{\theta W_t}{\theta - 1 s_t} \) characterizes the behavior of the firms while if \( \Xi_t < 0 \), the solution for the firms is to set \( y_t (i) = 0 \).

It is useful to write the condition \( \Xi_t \geq 0 \) in terms of the tax instruments. Recall that

\[
\Xi_t = (1 - \tau^d_t) E_t \frac{\beta u_{Ct+1}}{(1 + \tau^c_{t+1}) P_{t+1}} - (1 - \tau^d_t) \frac{u_{Ct}}{P_t (1 + \tau^c_t) R_{t+1}}
\]

using intertemporal conditions. If we also use the intratemporal condition, we obtain
\[ \Xi_t = (1 - \tau_t^d) \frac{u_{Ct}}{P_t(1 + \tau_t^n)R_{t+1}} = \frac{(1 - \tau_t^d)u_{1-Nt}}{(1 - \tau_t^n)W_t} \]

Thus, as the wage rate and the marginal utility of leisure are both positive, we can write the constraint as

\[ \frac{(1 - \tau_t^d)}{(1 - \tau_t^n)} \geq 0 \quad (12) \]

As \( G_t > 0 \), feasibility requires firms to produce positive quantities. Therefore, (12) must hold in equilibrium.

### 2.3.2 When prices are set in advance

We consider now an environment where firms set the prices one period in advance and can only sell output in period \( t \) at the previously chosen price. As of time \( t \), the firms are constrained in terms of the price at which they can sell, but are not constrained in terms of the quantity. Thus, at time \( t \), and given a previously chosen price, they do choose quantities to maximize profits. That problem is given by

\[
\max_{y(i)_t} \left[ P_t(i)y_t(i) - W_t \frac{y_t(i)}{s_t} \right] \Xi_t
\]

subject to

\[ 0 \leq y(i)_t \leq Y_t \left( \frac{P_t(i)}{P_t} \right)^{-\theta} \]

or

\[ 0 \leq y(i)_t \leq Y_t \]

since all firms are symmetric. The problem can be restated as

\[
\max_{y(i)_t} y_t(i) \left[ P_t(i) - \frac{W_t}{s_t} \right] \Xi_t
\]

The solution is to set \( y(i)_t = Y_t \) as long as \( \left[ P_t(i) - \frac{W_t}{s_t} \right] \Xi_t \geq 0 \), and \( y(i)_t = 0 \) otherwise. Thus, firms will satisfy demand as long as they do not make negative profits, and produce zero otherwise.
Firms at time $t - 1$ must choose $p(i)_t$, to maximize the value of the firm next period

$$E_{t-1}a(i)_t = \max_{p(i)_{t+1}} E_{t-1} d(i)_t \left(1 - \tau^d_t\right) \beta u_{Ct+1} + E_{t-1} \left[ \frac{a(i)_{t+1} \beta u_{Ct+1}}{P_{t+1} (1 + \tau^c_{t+1})} \right]$$

so, the optimal problem is to choose $P(i)_t$ to maximize,

$$E_{t-1} \left[ \frac{u_{Ct+1}}{P_{t+1} (1 + \tau^c_{t+1})} \right] (1 - \tau^d_t) y_t(i) \left( P_t(i) - \frac{W_t}{s_t} \right)$$

subject to (10).

The solution is given by

$$P_t(i) = P_t = \theta (\theta - 1) E_{t-1} \left[ \frac{W_t}{u_t s_t} \right]$$

where

$$u_t = \frac{\left(1 - \tau^d_t\right) u_{Ct+1}}{\left(1 + \tau^c_{t+1}\right) P_{t+1} y_t}.$$

As $G_t > 0$, feasibility requires firms to produce positive amounts of output every period. As we argued before, this is the case when $\left[ P_t(i) - \frac{W_t}{s_t} \right] \geq 0$.

Thus, in the SIA prices economy, feasibility means that

$$\left[ \frac{1}{w_t} - \frac{1}{s_t} \right] (1 - \tau^d_t) u_{1 - Nt} \geq 0$$

As the marginal utility is positive, the constraint is

$$\left[ \frac{1}{w_t} - \frac{1}{s_t} \right] (1 - \tau^d_t) \geq 0$$

(14)

2.4 Market clearing

Market clearing requires

$$C_t + G_t = Y_t = s_t N_t$$

$$B^h_t = B^g_t$$

$$Z^h_{t,t+1} = Z^g_{t,t+1}, \text{ for all possible states at } t + 1$$

where $B^g_t$ and $Z^g_{t,t+1}$ represent government debt, and

$$A(i)_t = 1$$
2.5 Implementability constraints

In order to define the set of allocations that can be decentralized as an equilibrium, it is useful to follow the dynamic Ramsey literature and use equilibrium conditions to substitute away prices and tax instruments and therefore reduce the dimension of the problem as much as possible. It turns out, however, that some, but not all tax instruments can be substituted away in this monopolistic competition model.

The period by period budget constraints of the households can be written, once we take into account that \( A(i)_t = 1, d(i)_t = D_t \) and \( c(i) = C_t \) for all \( i \), as

\[
M_0 + B_1^h + E_0 Z_1^h z_1 \leq W_0
\]

where \( W_0 = W_0 - \int_0^1 a(i) d_i \), and

\[
M_t + B_{t+1}^h + E_t Z_{t+1}^h z_{t+1} \leq M_{t-1} + R_t B_t^h + Z_t^h - P_{t-1} C_{t-1}(1 + \tau_{t-1}^c) + W_{t-1} N_{t-1}(1 - \tau_{t-1}^n) + D_{t-1}(1 - \tau_{t-1}^d)
\]

for \( t \geq 1 \). In order to avoid the well known time inconsistency problem of optimal monetary policy, we assume that \( W_0 = 0 \).

The present value budget constraint can be written if we multiply the equation by each state and period normalized contingent price \( Q_t = \prod_{j=0}^{z_j} \), take expectations and add for all \( t \)

\[
E_0 \sum_{t=0}^{\infty} Q_{t+1} \left( P_t C_t(1 + \tau_t^c) - W_t N_t(1 - \tau_t^n) - D_t(1 - \tau_t^d) + M_t \left( \frac{Q_t}{Q_{t+1}} - 1 \right) \right) = 0
\]

Using asset price equations and the cash-in-advance constraint, we obtain

\[
E_0 \sum_{t=0}^{\infty} \beta^t U_{Ct} \left( P_t C_t(1 + \tau_t^c) - \frac{W_t N_t(1 - \tau_t^n) + D_t(1 - \tau_t^d)}{R_{t+1}} \right) = 0
\]

Replacing the value of dividends and using the production function, we obtain

\[
E_0 \sum_{t=0}^{\infty} \beta^t U_{Ct} \left( C_t - \frac{w_t(1 - \tau_t^n)}{R_{t+1}(1 + \tau_t^c)} N_t + \frac{(1 - \tau_t^d)}{R_{t+1}(1 + \tau_t^c)} s_t N_t + \frac{w_t(1 - \tau_t^d)}{R_{t+1}(1 + \tau_t^c)} N_t \right) = 0
\]

Finally, using the intra-temporal marginal condition from the household problem we can eliminate \( R_{t+1}(1 + \tau_t^c) \) and obtain
\[ E_0 \sum_{t=0}^{\infty} \beta^t \left( U_{C_t} - u_{1-N_t}N_t + \frac{(1-\tau_d^t)}{(1-\tau^n_t)} u_{1-N_t}N_t \left( \frac{s_t}{w_t} - 1 \right) \right) = 0 \] (16)

### 2.5.1 Flexible prices

With flexible prices, real wages must satisfy

\[ \frac{s_t \theta - 1}{\theta} = w_t \] (17)

every period and state. If we replace these conditions on the budget constraint (16), we obtain

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left( U_{C_t} - u_{1-N_t}N_t + \frac{(1-\tau_d^t)}{(1-\tau^n_t)} u_{1-N_t}N_t \left( \frac{\theta - 1}{\theta - 1} \right) \right) = 0 \]

or

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left( U_{C_t} - u_{1-N_t}N_t \left( 1 - \frac{(1-\tau_d^t)}{(1-\tau^n_t)} \left( \frac{1}{\theta - 1} \right) \right) \right) = 0 \]

Thus, an allocation \( \{C_t, N_t\}_{t=0}^{\infty} \) is implementable if there exists a sequence \( \{(1-\tau_d^t)\}_{t=0}^{\infty} \) satisfying \( \frac{(1-\tau_d^t)}{(1-\tau^n_t)} \geq 0 \) for all \( t \) and all states and such that

\[ C_t + G_t = Y_t = s_tN_t \]

for all \( t \) and all states, and

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left( U_{C_t} - u_{1-N_t}N_t \right) \left( 1 - \frac{(1-\tau_d^t)}{(1-\tau^n_t)} \left( \frac{1}{\theta - 1} \right) \right) = 0 \]

hold.

### 2.5.2 SIA prices

The optimal pricing rule is

\[ 1 = E_t \left[ v_{t+1} \frac{\theta}{\theta - 1} \frac{w_{t+1}}{s_{t+1}} \right] \] (18)
for \( t \geq 0 \). We can follow Adao, Correia and Teles (2000) and write (18) as

\[
E_{t-1} \left[ \frac{(1 - \tau^d_t)u_{C_{t+1}}}{P_{t+1}(1 + \tau^d_{t+1})} y_t \frac{w_t}{s_t} - \frac{(1 - \tau^n_t)u_{C_{t+1}}}{P_{t+1}(1 + \tau^n_{t+1})} y_t \frac{\theta - 1}{\theta} \right] = 0.
\]

Using the law of iterated expectations and the intertemporal conditions we obtain

\[
E_{t-1} \left[ \left( y_t \frac{w_t}{s_t} - y_t \frac{\theta - 1}{\theta} \right) \frac{(1 - \tau^d_t)u_{C_t}}{R_{t+1}(1 + \tau^d_t)} \right] = 0
\]

since \( P_t \) is in the information set at \( t - 1 \). This can be written as

\[
\frac{\theta}{\theta - 1} E_{t-1} \left( N_t u_{1-N_t} \frac{(1 - \tau^d_t)}{(1 - \tau^n_t)} \right) = E_{t-1} \left( s_t \frac{w_t}{\theta} N_t u_{1-N_t} \frac{(1 - \tau^d_t)}{(1 - \tau^n_t)} \right)
\]

Using the law of iterated expectations

\[
\frac{\theta}{\theta - 1} E_0 \left( N_t u_{1-N_t} \frac{(1 - \tau^d_t)}{(1 - \tau^n_t)} \right) = E_0 \left( s_t \frac{w_t}{\theta} N_t u_{1-N_t} \frac{(1 - \tau^d_t)}{(1 - \tau^n_t)} \right)
\]

which implies

\[
\frac{1}{\theta - 1} E_0 \left( N_t u_{1-N_t} \frac{(1 - \tau^d_t)}{(1 - \tau^n_t)} \right) = E_0 \left( \frac{(1 - \tau^d_t)}{(1 - \tau^n_t)} u_{1-N_t} N_t \left( s_t \frac{w_t}{\theta} - 1 \right) \right)
\]

for \( t \geq 1 \).

On the other hand, recall that the life-time budget constraint is

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( U_{C_t} C_t - u_{1-N_t} N_t + \frac{(1 - \tau^d_t)}{(1 - \tau^n_t)} u_{1-N_t} N_t \left( s_t \frac{w_t}{\theta} - 1 \right) \right) = 0,
\]

so, using the equation above, it can be written as

\[
\left( U_{c_0} C_0 - u_{1-N_0} N_0 \left( 1 - \frac{(1 - \tau^d_0)}{(1 - \tau^n_0)} \left( s_0 \frac{w_0}{\theta - 1} \right) \right) \right) +
\]

\[
E_0 \sum_{t=1}^{\infty} \beta^t \left( U_{C_t} C_t - u_{1-N_t} N_t \left( 1 - \frac{1}{\theta - 1} \frac{(1 - \tau^d_t)}{(1 - \tau^n_t)} \right) \right) = 0
\]

Thus, an allocation \( \{ C_t, N_t \}_{t=0}^{\infty} \) is implementable in this SIA economy if there exist sequences \( \{(1 - \tau^d_t) / (1 - \tau^n_t), w_t\}_{t=0}^{\infty} \) satisfying \( \left[ \frac{1}{w_t} - \frac{1}{s_t} \right] \frac{(1 - \tau^d_t)}{(1 - \tau^n_t)} \geq 0 \) for all \( t \) and all states, and such that
\[
E_{t-1} \left[ \left( \frac{w_t}{s_t} - \frac{\theta - 1}{\theta} \right) \frac{y_t(1 - \tau^d_t)u_{Ct}}{R_{t+1}(1 + \tau^c_t)} \right] = 0
\]
for all \( t \geq 1 \),

\[
C_t + G_t = Y_t = s_tN_t
\]
for all \( t \) and all states, and

\[
\left( U_{C0}C_0 - u_{1-N0}N_0 \left( 1 - \frac{(1 - \tau^d_0)}{(1 - \tau^d_0)} \left( \frac{s_0}{w_0} - 1 \right) \right) \right) +
\]

\[
E_0 \sum_{t=1}^{\infty} \beta^t \left( U_{Ct}C_t - u_{1-Nt}N_t \left( 1 - \frac{1}{\theta - 1} \left( \frac{1 - \tau^d_t}{1 - \tau^n_t} \right) \right) \right) = 0
\]
hold.

The only difference between this last implementability constraint and the one of the flexible prices is the additional choice of the real wage in the first period.

On the other hand, note that the real wages of the flexible prices economy trivially satisfy the optimal pricing rule of the SIA prices economy. Indeed, real wages are constrained by the constant mark-up and the productivity shock on a state by state basis in the flexible prices economy while they must satisfy a single expected value condition in the SIA prices economy. Thus, the definition of the set of implementable allocations in the flexible prices economy appears to be stricter than in the SIA prices economy.

The next lemma formally proves this intuition.

**Lemma 1** If an allocation is implementable in the flexible prices economy, then it is also implementable in the SIA prices economy.

**Pf:** If \( \{C_t^*, N_t^*\}_{t=0}^{\infty} \) can be implemented in the flexible prices economy, then there exists \( \{(1 - \tau^d_t)/(1 - \tau^n_t)\}_{t=0}^{\infty} \) satisfying \( \{(1 - \tau^d_t)/(1 - \tau^n_t)\}_{t=0}^{\infty} \geq 0 \) for all \( t \), and all states, and

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( U_{Ct}C_t^* - u_{1-Nt^*}N_t^* \left( 1 - \frac{(1 - \tau^d_t)}{(1 - \tau^n_t)} \left( \frac{1}{\theta - 1} \right) \right) \right) = 0
\]
Now, let $w_t = \frac{\theta - 1}{\theta} s_t$ for all $t$ and all $\sigma^t$. Then, clearly

$$\left[ \frac{1}{w_t} - \frac{1}{s_t} \right] (1 - \tau^d_t) = \frac{1}{\theta - 1} \left( \frac{1 - \frac{\sigma^t}{w_t}}{1 - \tau^d_t} \right) \geq 0,$$

since $\theta > 1$. This real wage trivially satisfies the optimal pricing equation. Finally, note that $\frac{1}{\theta - 1} = \frac{\theta}{\theta - 1} - 1 = \frac{s_t}{w_t} - 1$. Replacing on the implementability constraint in the first period we obtain

$$\left( U_{C^t \cdot} C_0^* - u_{1-N0}^* N_0^* \left( 1 - \frac{(1 - \frac{\tau^d_0}{1 - \tau^d_0})}{(1 - \frac{\tau^d_0}{1 - \tau^d_0})} \right) \right) +$$

$$E_0 \sum_{t=1}^{\infty} \beta^t \left( U_{C^t \cdot} C_t^* - N_t^* u_{1-N_t}^* \left( 1 - \frac{1}{\theta - 1} \left( \frac{1 - \tau^d_t}{1 - \tau^d_t} \right) \right) \right) = 0$$

As the allocation also satisfies feasibility, then it can be implemented in the SIA prices economy. QED.

This Lemma shows that it is feasible, in the SIA prices economy, to use monetary policy to replicate the real wage that would prevail under flexible prices. The natural question now is if the additional degree of freedom allows the planner in the SIA to implement allocations that cannot be implemented in the flexible prices economy. The next proposition shows that this is not the case.

**Proposition 2** If an allocation is implementable in the SIA economy, then it is also implementable in the flexible prices economy.

**Pf:** If $\{C_t, N_t\}_{t=0}^\infty$ can be implemented in the SIA economy, then there exists $\left\{ \frac{1 - \tau^d_t}{1 - \tau^d_t}, w_t \right\}_{t=0}^\infty$ satisfying $\left[ \frac{1}{w_t} - \frac{1}{s_t} \right] (1 - \tau^d_t) \geq 0$ for all $t$,

$$\left( U_{C^0 \cdot} C_0^* - u_{1-N0}^* N_0^* \left( 1 - \frac{(1 - \frac{\tau^d_0}{1 - \tau^d_0})}{(1 - \frac{\tau^d_0}{1 - \tau^d_0})} \right) \right) +$$

$$E_0 \sum_{t=1}^{\infty} \beta^t \left( U_{C^t \cdot} C_t^* - N_t^* u_{1-N_t}^* \left( 1 - \frac{1}{\theta - 1} \left( \frac{1 - \tau^d_t}{1 - \tau^d_t} \right) \right) \right) = 0$$

and the optimal pricing rule
\[ \frac{1}{\theta - 1} E_0 \left( N_t u_{1-N_t} \frac{1 - \tau_t^d}{1 - \tau_t^n} \right) = E_0 \left( \frac{s_t}{w_t} - 1 \right) \]

Let \( \tau_t^d \) and \( \tau_t^n \) be defined by \( \left[ \frac{s_t}{w_t} - 1 \right] \frac{(1 - \tau_t^d)}{(1 - \tau_t^n)} = \frac{1}{\theta - 1} \frac{(1 - \tau_t^d)}{(1 - \tau_t^n)} \) for all \( t \geq 0 \).

Then, \( \frac{1}{w_t} \left[ \frac{1 - \tau_t^d}{1 - \tau_t^n} \right] \geq 0 \) for all \( t \), we obtain \( \left[ \frac{s_t}{w_t} - 1 \right] \frac{(1 - \tau_t^d)}{(1 - \tau_t^n)} \geq 0 \), since \( s_t > 0 \). Therefore, \( \frac{1}{\theta - 1} \frac{(1 - \tau_t^d)}{(1 - \tau_t^n)} \geq 0 \), which, since \( \theta > 1 \), implies \( \frac{(1 - \tau_t^d)}{(1 - \tau_t^n)} \geq 0 \). In addition, using the definition of \( \frac{(1 - \tau_t^d)}{(1 - \tau_t^n)} \) in the optimal pricing rule, we obtain

\[ E_0 \left( \frac{s_t}{w_t} - 1 \right) N_t u_{1-N_t} \frac{(1 - \tau_t^d)}{(1 - \tau_t^n)} = \frac{1}{\theta - 1} E_0 \left( N_t u_{1-N_t} \frac{(1 - \tau_t^n)}{(1 - \tau_t^d)} \right) \]

Thus, the implementability above can be written as

\[ \left( U_{C^0} \cdot C_0^* - u_{1-N_0} \cdot N_0^* + \frac{1}{\theta - 1} \frac{(1 - \tau_0^d)}{\tau_0^n} u_{1-N_0} \cdot N_0 \right) + \]

\[ E_0 \sum_{t=1}^{\infty} \beta^t \left( U_{C^t} \cdot C_t^* - u_{1-N_t} \cdot N_t^* + \frac{1}{\theta - 1} \left( N_t^* u_{1-N_t} \cdot \frac{(1 - \tau_t^d)}{(1 - \tau_t^n)} \right) \right) = 0 \]

or

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left( U_{C^t} \cdot C_t^* - u_{1-N_t} \cdot N_t^* \left( 1 - \frac{1}{\theta - 1} \frac{(1 - \tau_t^d)}{(1 - \tau_t^n)} \right) \right) = 0 \]

As the allocation also satisfies feasibility, then it can be implemented in the flexible prices economy. QED.

The proposition shows that any allocation that can be implemented in the sticky prices economy can always be implemented using a policy that replicates the flexible prices wages. Thus means that although in fact the government can use, in the SIA economy, monetary policy to affect the allocation, there are other policy instruments that operate in exactly the same way as monetary policy does. Indeed, the proof of the proposition highlights the equivalence between monetary policy induced real wage departures from the flexible price wages - gaps - and other fiscal instruments - labor income taxes and profit taxes in that case.
This principle applies for any feasible allocation, so it suggests that there may be multiple policies that implement a given allocation. In particular, one may want to characterize the policies that implement the Ramsey allocation. We now turn to the discussion of polices.

### 2.6 Decentralization

So far, we focused our discussion on the sets of implementable allocations. We now want to discuss, given a particular allocation, the policies that decentralize it.

#### 2.6.1 The flexible prices economy

Given an implementable allocation \( \{C^*_t, N^*_t\}_{t=0}^\infty \), the sequence \( \{\tau^c_t, \tau^n_t, \tau^d_t, R_{t+1}\}_{t=0}^\infty \) is a policy that decentralizes that allocation for the flexible prices economy, if the following conditions hold

\[
C_t + G_t = Y_t = s_t N_t
\]

\[
\frac{u_{1-N_t^*}}{u^{C^*_t}} = s_t \theta - 1 \left( \frac{1 - \tau^n_t}{R_{t+1}(1 + \tau_c^t)} \right)
\]

\[
P_tC^*_t(1 + \tau^c_t) = M_t
\]

\[
\frac{(1 - \tau^n_t^d)(1 - \tau^n_t)}{(1 - \tau^n_t)} \geq 0
\]

for \( t \geq 0 \) and all states, were \( \frac{u_{1-N_t^*}}{u^{C^*_t}} \) is the MRS evaluated at \((C^*_t, N^*_t)\), and

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( U_{C^*_t} C^*_t - u_{1-N_t^*} N^*_t \left( \frac{1 - (1 - \tau^n_t^d)}{(1 - \tau^n_t)^{\frac{1}{\theta - 1}}} \right) \right) = 0.
\]

First, note that if the allocation is implementable, then the first equation must be satisfied in every period and every state. In addition, there exists a sequence \( \frac{(1 - \tau^n_t^d)}{(1 - \tau^n_t)} \geq 0 \) such that the last condition hold. Note that only the ratio is relevant for the last condition. Thus, we consider the income tax rate as a free instrument and leave the profit tax rate to be pinned down so as to satisfy the last condition. Then, if \( \sigma_t \) can take \( N \) different values, (19) is a set of \( N \) equations that uniquely determine the \( N \) ratios \( \frac{(1 - \tau^n_t^d)}{(1 - \tau^n_t)} \).
Thus, this three instruments are equivalent. Thus, let $R_{t+1} = 1^5$. Clearly there is a continuum of values for the remaining two policy instruments that implement any given allocation.

To focus the discussion that follows in monetary policy, we assume that $\tau _t(\sigma ^t) = 0$. Given the allocation and any value for $M_t$, the nominal interest rate $R_{t+1}$ is uniquely pinned down by future expected monetary policy according to

$$
\frac{u_{C_t^*}C_t^*}{M_t} = R_{t+1}E_t \left[ \frac{\beta u_{C_{t+1}^*}C_{t+1}^*}{M_{t+1}} \right]
$$

or

$$
\frac{u_{C_t^*}C_t^*}{\mu(\sigma ^t)} = \frac{R_{t+1}}{\mu(\sigma ^t)}E_t \left[ \frac{\beta u_{C_{t+1}^*}C_{t+1}^*}{\delta(\sigma ^{t+1})} \right]
$$

while the price level is determined by $M_t$ according to the money demand equation

$$
P_tC_t^* = M_t
$$

which can be written as

$$
P_t(\sigma ^t)C_t^*(\sigma ^t) = M_{t-1}\mu(\sigma ^{t-1})\delta(\sigma ^t)
$$

for all $\sigma ^t$, were the condition

$$
E_{t-1}(\delta(\sigma ^t)) = 1
$$

must hold. These $N + 1$ equations solve for the two sequences $P_t(\sigma ^t)$ and $\delta(\sigma ^t)^6$. Then, given $N - 1$ values for $\delta(\sigma ^t)$, the sequence $P_t(\sigma ^t)$ gets fully determined.

These equations make clear that the expected component of monetary policy uniquely pins down the value for the nominal interest rate, while the unexpected component determines the price level. They also make clear the notion of monetary neutrality in this flexible prices model, since the choices

\footnote{It is convenient to consider this case, since this equivalence between $R_{t+1}$ and $(1 + \tau _t^\xi)$ is not robust to cash-credit or transactions technologies extensions of the model. See Correia, Nicolini and Teles (2001a).}

\footnote{If we do not set $\tau _t^\xi(\sigma ^t) = 0$, then the quantity of money and the consumption tax rate are equivalent instruments.}
for the $\delta(\sigma^t)$ only affect the price levels, without affecting the allocation. Note, however, that in this rational expectations model this money neutrality exercise, requires, for consistency that when the value for $\delta(\sigma^t)$ is changed in a given state, it has to be changed in some other state, in order to satisfy the constraint $E_{t-1}(\delta(\sigma^t)) = 1$. Otherwise, the expected component of money growth will change, a policy exercise associated to the concept of superneutrality. In fact, in this model, money is not superneutral, since changes in $\mu(\sigma^t)$ affect the nominal interest rate and therefore the marginal rate of substitution between consumption and leisure.

2.6.2 SIA prices

Given an implementable allocation $\{C^*_t, N^*_t\}_{t=0}^\infty$, the sequence $\{\tau^c_t, \tau^n_t, \tau^d_t, R_{t+1}, w_t\}_{t=0}^\infty$ is a policy that decentralizes that allocation for the SIA prices economy, if the following conditions hold

$$C_t + G_t = Y_t = s_t N_t$$

$$P_tC^*_t(1 + \tau^c_t) = M_{t-1}\mu(\sigma^{t-1})\delta(\sigma^t)$$

$$\frac{u_{1-N^*_t}}{u_{C^*_t}} = w_t \frac{(1 - \tau^n_t)}{R_{t+1}(1 + \tau^c_t)}$$

for all $t$ and all states

$$P_t = \frac{\theta}{(\theta - 1)} E_{t-1} \left[ \frac{W_t}{v_t s_t} \right]$$

for $t \geq 1$}

$$\left[ \frac{1}{w_t} - \frac{1}{s_t} \right] \frac{(1 - \tau^d_t)}{(1 - \tau^n_t)} \geq 0$$

for all $t$ and all states, the implementability constraint

$$\left( U_{C^*_0}^* - u_{1-N^*_0} N^*_0 + \frac{(1 - \tau^d_0)}{(1 - \tau^n_0)} u_{1-N^*_0} N^*_0 \left( \frac{s_0}{w_0} - 1 \right) \right) +$$

$$E_0 \sum_{t=1}^{\infty} \beta^t \left( U_{C^*_t}^* - u_{1-N^*_t} N^*_t + \frac{1}{\theta - 1} \left( N^*_t u_{1-N^*_t} \frac{(1 - \tau^d_t)}{(1 - \tau^n_t)} \right) \right) = 0$$

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As before, the first equation is satisfied since the allocation is implementable and the profit tax adjust to changes in the income tax rate so as to maintain the last two conditions. Similarly, the nominal interest rate $R_{t+1}$ is pinned down by future expected monetary policy according to

$$u_{Ct}C_t^* = \frac{R_{t+1}}{\mu(\sigma^t)}E_t \left[ \frac{\beta u_{C_{t+1}}C_{t+1}^*}{\delta(\sigma^{t+1})} \right]$$

Thus, for any $t \geq 1$, making explicit the dependence of the variables in the states, we have the following equations

$$P_t(\sigma^{t-1})C_t^*(\sigma^{t-1}, \sigma_t)(1 + \tau^c_t(\sigma^{t-1}, \sigma_t)) = M_t(\sigma^{t-1})\delta(\sigma^{t-1}, \sigma_t) \tag{25}$$

$$\frac{u_{1-Nt^*}}{u_{Ct^*}}(\sigma^{t-1}, \sigma_t) = \frac{W_t(\sigma^{t-1}, \sigma_t)}{P_t(\sigma^{t-1})} \frac{(1 - \tau^w_t(\sigma^{t-1}, \sigma_t))}{R_{t+1}(\sigma^{t-1}, \sigma_t)(1 + \tau^c_t(\sigma^{t-1}, \sigma_t))}$$

for all $\sigma_t$ and the optimal pricing rule, that can be written as

$$P_t(\sigma^{t-1}) = \frac{\theta}{(\theta - 1)}E_{t-1} \left[ y_t(\frac{1-\tau^w_t}{R_{t+1}(1+\tau^c_t)}) (\sigma^{t-1}, \sigma_t) \frac{W_t(\sigma^{t-1}, \sigma_t)}{E_{t-1} \left[ y_t(\frac{1-\tau^w_t}{R_{t+1}(1+\tau^c_t)}) (\sigma^{t-1}, \sigma_t) \right]} \right]$$

Note that, if at any period there is a number $N$ of states, the first set of equations involve $2N$ equations, while the last one is a single condition. In addition, the condition

$$E_{t-1}(\delta(\sigma^t)) = 1$$

must also hold. Thus, we have $2N + 2$ conditions to solve for $P_t(\sigma^{t-1})$, and $W_t(\sigma^{t-1}, \sigma_t), \delta(\sigma^{t-1}, \sigma_t), \tau^w_t(\sigma^{t-1}, \sigma_t), \tau^c_t(\sigma^{t-1}, \sigma_t), R_{t+1}(\sigma^{t-1}, \sigma_t)$ and $\mu(\sigma^{t-1}, \sigma_t)$ for all $\sigma_t$.

As before, the consumption tax, the nominal interest rate and the labor tax rate are equivalent instruments, so we set $\tau^c_t(\sigma^{t-1}, \sigma_t) = 0$ and $R_{t+1}(\sigma^{t-1}, \sigma_t) = 1$. Then, we have the following set of equations

$$P_t(\sigma^{t-1})C_t^*(\sigma^{t-1}, \sigma_t) = M_t(\sigma^{t-1})\delta(\sigma^{t-1}, \sigma_t) \tag{26}$$

$$\frac{u_{1-Nt^*}}{u_{Ct^*}}(\sigma^{t-1}, \sigma_t) = \frac{W_t(\sigma^{t-1}, \sigma_t)}{P_t(\sigma^{t-1})} (1 - \tau^w_t(\sigma^{t-1}, \sigma_t))$$

for all $\sigma_t$ and
\[
P_t(\sigma^{t-1}) = \frac{\theta}{(\theta - 1)} E_{t-1} \left[ \frac{y_t(1 - \tau_t^d)u_{Ci}^t(\sigma^{t-1}, \sigma_t)}{E_{t-1} [y_t(1 - \tau_t^d)u_{Ci}^t(\sigma^{t-1})] s_t(\sigma^{t-1}, \sigma_t)]} \right]
\]

which constitute a system of \(2N + 2\) equations to solve for \(P_t(\sigma^{t-1})\), and \(W_t(\sigma^{t-1}, \sigma_t), \delta(\sigma^{t-1}, \sigma_t), \tau_t^n(\sigma^{t-1}, \sigma_t)\) for all \(\sigma_t\).

First, note that for any given value of \(\sigma_t = \tilde{\sigma}_t\), we can write the first set of equations as

\[
\frac{P_t(\sigma^{t-1})C_t^*(\sigma^{t-1}, \sigma_t)}{P_t(\sigma^{t-1})C_t^*(\sigma^{t-1}, \tilde{\sigma}_t)} = \frac{M_t(\sigma^{t-1})\delta(\sigma^{t-1}, \sigma_t)}{M_t(\sigma^{t-1})\delta(\sigma^{t-1}, \tilde{\sigma}_t)}
\]

or

\[
\delta(\sigma^{t-1}, \tilde{\sigma}_t)C_t^*(\sigma^{t-1}, \sigma_t) = C_t^*(\sigma^{t-1}, \tilde{\sigma}_t)\delta(\sigma^{t-1}, \sigma_t)
\]

for all \(\sigma_t \neq \tilde{\sigma}_t\). Thus, given an allocation these \(N - 1\) equations can be combined with

\[
E_{t-1}(\delta(\sigma^{t-1}, \sigma_t)) = 1
\]

to find the unique solution for the \(N\) policy instruments \(\delta(\sigma^{t-1}, \sigma_t)\). Thus, money is not neutral anymore.8

Given this solution, the equilibrium value for the price level is determined by any of the conditions

\[
P_t(\sigma^{t-1})C_t^*(\sigma^{t-1}, \sigma_t) = \bar{M}_t(\sigma^{t-1})\delta(\sigma^{t-1}, \sigma_t)
\]

Then, the remaining \(N + 1\) equations

\[
\frac{u_{1-Nt^*}}{u_{Ci^*}}(\sigma^{t-1}, \sigma_t) = \frac{W_t(\sigma^{t-1}, \sigma_t)}{P_t(\sigma^{t-1})}(1 - \tau_t^n(\sigma^{t-1}, \sigma_t))
\]

7 The profit tax enters affecting the weights of the expected value. However, we do not consider it as an instrument in this discussion, as we mentioned above, since it is used to satisfy the implementability constraint.

8 Setting the consumption tax rates at zero seems less innocent an assumption in this case, since it appears as an equivalent instrument to monetary policy. The reason why we ignore this possibility resides in the fact that we do not have strong reasons to believe that price stickiness is either a gross or net of taxes phenomena. The ability of consumption tax rates to pin down allocations in this sticky prices model crucially depends on that feature.
for all $\sigma_t$ and

$$P_t(\sigma^{t-1}) = \frac{\theta}{(\theta - 1)} E_{t-1} \left[ \frac{y_t(1 - \tau_t) u_{Ct}(\sigma_t^{t-1}, \sigma_t) W_t(\sigma_t^{t-1}, \sigma_t)}{E_{t-1} [y_t(1 - \tau_t) u_{Ct}](\sigma_t^{t-1}) s_t(\sigma_t^{t-1}, \sigma_t)} \right]$$

determine the $2N$ solutions for $W_t(\sigma^{t-1}, \sigma_t)$ and $\tau^n_t(\sigma^{t-1}, \sigma_t)$ for all $\sigma_t$. Clearly, there are multiple solutions.

This solution multiplicity can be interpreted as a tax neutrality result, similar to the neutrality of money in the flexible prices economy. Note that if we use the first $(N)$ equations to eliminate the $(N)$ wages, we obtain a single equation restricting the values for the tax rates that decentralize a given allocation

$$1 = \frac{\theta}{(\theta - 1)} E_{t-1} \left[ \frac{y_t(1 - \tau_t) u_{Ct}(\sigma_t^{t-1}, \sigma_t)}{y_t(1 - \tau_t) u_{Ct}} \frac{1}{(1 - \tau_t^n(\sigma_t^{t-1}, \sigma_t))} \right]$$

It is interesting to note the symmetric nature of the result. Increasing the tax rate in a given state, only affects the nominal wage of that state. However, the exercise requires the same tax rate to be changed in other states, so as to satisfy the constraint above. Thus, alternative income tax functions satisfying $E_{t-1}(\delta(\sigma_t)) = 1$ only affect the distribution of nominal wages $W_t(\sigma_t^{t-1}, \sigma_t)$, but are in all other respects, neutral for the determination of the equilibrium. Recall that in the flexible prices economy, alternative values for $\delta(\sigma_t)$ satisfying $E_{t-1}(\delta(\sigma_t)) = 1$ only affect the distribution of price levels $P_t(\sigma_t)$, but are neutral.

There is, however, a major difference between the neutrality of fiscal instruments in this case and the neutrality of money in the flexible prices economy. This difference arises due to the feasibility constraint

$$\left[ \frac{1}{w_t} - \frac{1}{s_t} \right] \frac{(1 - \tau^n_t)}{(1 - \tau^n_t)} \geq 0$$

Thus, our neutrality results on fiscal instruments are not global results, since they hold only to the extent that the feasibility constraint holds.

Finally, note that while money is not neutral when prices are set in advance, it is superneutral. This is so, since $R_{t+1}$ is an instrument equivalent to the labor income tax rate, so everything we said above regarding the income tax rate extends to the nominal interest rate (and, for that matter, to the consumption tax rate).
In summary, in both the flexible and SIA prices economies, there are multiple policies that decentralize a given allocation. The multiplicity arises because of two different reasons. First, in both economies there are neutral instruments. Thus, the particular values of those neutral instruments are irrelevant. Second, in both economies there are equivalent instruments. Thus, different combinations of non-neutral but equivalent instruments decentralize any given allocation.

It turns out, however, that the existence of neutral instruments is not robust to small perturbations of the models. We address this issue in the next section, where we solve a model in which a fraction of the firms sets prices in advance, while the rest set state contingent prices. This model nests the two extremes that we analyzed in this section.

3 Economies with heterogenous price setting

The previous section showed that the optimal allocation is independent of the price setting restrictions. However, it also showed that the way policies affect allocations is very sensitive to these restrictions. A priori, this may suggest that optimal policies may be very sensitive to the price setting frictions in a way that makes general results hard to obtain. In this section we show that this a priori vision is incorrect, and a single optimal fiscal and monetary policy can be characterized independently of the degree of price stickiness.

We modify the model and assume that a fraction $\alpha$ of the firms must set prices in advance while a fraction $1 - \alpha$ can set state contingent prices. Given symmetry, all sticky firms will set the price $P^S_t$ and all flexible firms set the common price $P^F_t$. The price level is given by

$$P_t = \left[ \alpha P^S_t (1-\theta) + (1 - \alpha) P^F_t (1-\theta) \right]^{\frac{1}{1-\theta}}$$

Dividing by $W_t$

$$\frac{1}{w_t} = \left[ \alpha \left( \frac{1}{w_t^S} \right)^{(1-\theta)} + (1 - \alpha) \left( \frac{1}{w_t^F} \right)^{(1-\theta)} \right]^{\frac{1}{1-\theta}}$$

where $w_t^S = \frac{W_t}{P_t^S}$ and $w_t^F = \frac{W_t}{P_t^F}$. The pricing rule of the firms that set state contingent prices is

$$w_t^F = \frac{\theta - 1}{\theta} s_t$$
while
\[ E_{t-1} \left[ u_{Ct} \frac{w_t(1 - \tau_d^t)}{R_{t+1}(1 + \tau_c^t) w_t^S} \left( \frac{\theta - 1}{\theta} - \frac{w_t^S}{s_t} \right) y_t^S \right] = 0 \]
describes the pricing rule of firms that set prices one period in advance. The variable \( y_t^S \) represents the output of firms that set prices one period in advance. The problem of the consumer is the same as before, so the intertemporal budget constraint is the same
\[ E_0 \sum_{t=0}^{\infty} \beta^t U_{Ct} \left( C_t - \frac{w_t(1 - \tau_d^t)}{R_{t+1}(1 + \tau_c^t) W_t R_{t+1}} N_t - \frac{D_t(1 - \tau_d^t)}{W_t R_{t+1}} w_t \right) = 0 \]
In this economy, total dividends are
\[ D_t = \alpha y_t^S \left( P_t^S - \frac{W_t}{s_t} \right) + (1 - \alpha) y_t^F \left( P_t^F - \frac{W_t}{s_t} \right) \]
Note that
\[ E_0 \sum_{t=0}^{\infty} \beta^t U_{Ct} \left( 1 - \frac{\tau_d^t}{1 + \tau_c^t} \frac{D_t}{W_t R_{t+1}} \right) \]
\[ = E_0 \sum_{t=0}^{\infty} \beta^t U_{Ct} \left( 1 - \frac{\tau_d^t}{1 + \tau_c^t} \frac{w_t}{R_{t+1}} \right) \left( \alpha y_t^S \left( \frac{1}{w_t^S} - \frac{1}{s_t} \right) + (1 - \alpha) y_t^F \frac{1}{\theta - 1} \frac{1}{s_t} \right) \]
The pricing rule of the sticky firms can be written as
\[ E_0 \left[ u_{Ct} \frac{w_t(1 - \tau_d^t)}{R_{t+1}(1 + \tau_c^t) w_t^S} y_t^S \right] = E_0 \left[ u_{Ct} \frac{w_t(1 - \tau_d^t)}{R_{t+1}(1 + \tau_c^t) \theta - 1} \frac{1}{s_t} y_t^S \right] \]
so, replacing above
\[ E_0 \sum_{t=0}^{\infty} \beta^t U_{Ct} \left( 1 - \frac{\tau_d^t}{1 + \tau_c^t} \frac{w_t}{R_{t+1}} \frac{D_t}{W_t} \right) \]
\[ = E_0 \sum_{t=0}^{\infty} \beta^t U_{Ct} \left( 1 - \frac{\tau_d^t}{1 + \tau_c^t} \frac{w_t}{R_{t+1}} \frac{1}{s_t} \right) \left( \frac{1}{\theta - 1} \right) \left( \alpha y_t^S + (1 - \alpha) y_t^F \right) \]
Thus, the implementability conditions become
\[ E_0 \sum_{t=0}^{\infty} \beta^t U_{Ct} \left( C_t - \frac{w_t(1 - \tau_d^t)}{R_{t+1}(1 + \tau_c^t) W_t} N_t - \frac{D_t(1 - \tau_d^t)}{W_t R_{t+1}} w_t \right) \left( \frac{1}{\theta - 1} \right) \left( \alpha y_t^S + (1 - \alpha) y_t^F \right) \]
\[
\frac{u_{1-N_t}}{u_{Ct}} = w_t \frac{(1 - \tau_t^S)}{R_{t+1}(1 + \tau_t^F)} \tag{31}
\]

\[c_t^S + g_t^S = y_t^s \tag{32}\]

\[c_t^F + g_t^F = y_t^F \tag{33}\]

\[\frac{c_t^S}{c_t^F} = \left(\frac{w_t^F}{w_t^S}\right)^{-\theta} \tag{34}\]

\[\frac{g_t^S}{g_t^F} = \left(\frac{w_t^F}{w_t^S}\right)^{-\theta} \tag{35}\]

\[C_t = \left[ \alpha c_t^S \theta + (1 - \alpha) c_t^F \theta \right]^{\frac{1}{\theta}}, \theta > 1. \tag{36}\]

\[G_t = \left[ \alpha g_t^S \theta + (1 - \alpha) g_t^F \theta \right]^{\frac{1}{\theta}}, \theta > 1. \tag{37}\]

\[
\frac{1}{w_t} = \left[ \alpha \left( \frac{1}{w_t^S} \right)^{(1-\theta)} + (1 - \alpha) \left( \frac{1}{w_t^F} \right)^{(1-\theta)} \right]^{\frac{1}{1-\theta}}
\]

\[\alpha y_t^s + (1 - \alpha) y_t^F = s_t N_t \tag{39}\]

\[w_t^F = \frac{\theta - 1}{s_t} \tag{40}\]

\[E_{t-1} \left[ \frac{w_t(1 - \tau_t^S)}{R_{t+1}(1 + \tau_t^F)} \frac{1}{w_t^S} g_t^S \right] = E_{t-1} \left[ \frac{\theta}{\theta - 1} u_{Ct} \frac{w_t(1 - \tau_t^F)}{R_{t+1}(1 + \tau_t^F)} \frac{1}{s_t} g_t^S \right] \tag{41}\]

**Lemma 3** In a competitive equilibrium with taxes and \( \alpha \in (0, 1) \) \( w_t^S \neq w_t^F \Leftrightarrow C_t + G_t < s_t N_t \), and \( w_t^S = w_t^F \Leftrightarrow C_t + G_t = s_t N_t \).

**Corollary 4** For any \( 0 < \alpha < 1 \), the sets of allocations that can be implemented with \( w_t^S = w_t^F \) and with \( w_t^S \neq w_t^F \) are mutually exclusive.
Proof: As \( \frac{\theta - 1}{\theta} < 1 \),

\[
\begin{align*}
\alpha \left( c^S_t \right)^{\frac{\theta - 1}{\theta}} + (1 - \alpha) \left( c^F_t \right)^{\frac{\theta - 1}{\theta}} &< \alpha c^S_t + (1 - \alpha) c^F_t \iff w^S_t \neq w^F_t \\
\alpha \left( c^S_t \right)^{\frac{\theta - 1}{\theta}} + (1 - \alpha) \left( c^F_t \right)^{\frac{\theta - 1}{\theta}} &= \alpha c^S_t + (1 - \alpha) c^F_t \iff w^S_t = w^F_t
\end{align*}
\]

and

\[
\begin{align*}
\alpha \left( g^S_t \right)^{\frac{\theta - 1}{\theta}} + (1 - \alpha) \left( g^F_t \right)^{\frac{\theta - 1}{\theta}} &< \alpha g^S_t + (1 - \alpha) g^F_t \iff w^S_t \neq w^F_t \\
\alpha \left( g^S_t \right)^{\frac{\theta - 1}{\theta}} + (1 - \alpha) \left( g^F_t \right)^{\frac{\theta - 1}{\theta}} &= \alpha g^S_t + (1 - \alpha) g^F_t \iff w^S_t = w^F_t
\end{align*}
\]

Then, using (36) and (37)

\[
\begin{align*}
C_t + G_t &< \alpha c^S_t + (1 - \alpha) c^F_t + \alpha g^S_t + (1 - \alpha) g^F_t \iff w^S_t \neq w^F_t \\
C_t + G_t &= \alpha c^S_t + (1 - \alpha) c^F_t + \alpha g^S_t + (1 - \alpha) g^F_t \iff w^S_t = w^F_t
\end{align*}
\]

or, using (32), (33) and (39)

\[
\begin{align*}
C_t + G_t &< \alpha y^S_t + (1 - \alpha) y^F_t = s_t N_t \iff w^S_t \neq w^F_t \\
C_t + G_t &= \alpha y^S_t + (1 - \alpha) y^F_t = s_t N_t \iff w^S_t = w^F_t
\end{align*}
\]

This lemma implies that the aggregates when \( w^S_t \neq w^F_t \) do not belong to the production possibilities frontier, while aggregates when \( w^S_t = w^F_t \) do.

**Proposition 5** If the social welfare function is increasing on \( C_t \) and decreasing on \( N_t \), the optimal allocation will exhibit \( w^S_t = w^F_t \).

**Proof.** When \( w^S_t = w^F_t \), the equilibrium conditions for the aggregates \( \{C_t, N_t, \}_{t=0}^{\infty} \) are given by (31), the intertemporal budget constraint that, using equation (39) becomes

\[
\begin{align*}
E_0 \sum_{t=0}^{\infty} \beta^t U_{C_t} \left( C_t - \frac{w_t(1 - \tau^n_t)}{R_t + 1} \right) N_t - \frac{(1 - \tau^d_t) w_t}{(1 + \tau^{\theta}_t) R_t + 1} \left( \frac{1}{\theta - 1} \right) N_t
\end{align*}
\]

and the aggregate consistency condition

\[ C_t + G_t = s_t N_t \]

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When $w_t^S \neq w_t^F$, the equations are the same except for the aggregate consistency condition that becomes

$$C_t + G_t < s_tN_t$$

By setting $w_t^S = w_t^F$, the planner maximizes aggregate consumption given a value for aggregate labor. ■

Thus, if, for instance, we assume a benevolent government, then the optimal allocation will exhibit no distortion between the different varieties. This is an application of the well-known result of Diamond and Mirlees, since the varieties are intermediate inputs that enter in a constant returns to scale fashion into the "production" of the final good.

Note also that by setting $w_t^F = w_t^S$, we obtain the same implementability constraint as we obtained in the two extreme cases of the previous section. Thus, in this mixed economy, the government can attain allocations that cannot be implemented in the two extremes. The reason being that monetary policy is an instrument to distort the relative price between final consumption goods that is not available in the two extremes. However, this distortion is never used by the planner if social preferences are increasing in $C_t$ and decreasing in $N_t$. Thus, the Ramsey allocation in the mixed economy, for any degree of stickiness is the same as in the two extremes.

Note also that once we focus on allocations that satisfy $w_t^S = w_t^F$, the equilibrium conditions become independent of $\alpha$. The following corollaries follow.

**Corollary 6** The optimal allocation is independent of $\alpha$.

**Corollary 7** (Adao, Correia and Teles,(1999)) The optimal monetary policy does not depend on $\alpha$. There is a unique optimal monetary policy that involves setting the money supply so that the price does not react to contemporaneous information and $w_t^S = w_t^F$.

### 3.1 Decentralization

We now discuss optimal policies in this mixed economy. The implementability conditions described above (30) to (41) are the ones to be used to show how any given allocation determines the policies that implement it. First, note that $R_{t+1}$ and $(1 + \tau_t c_t)$ are equivalent instruments. So, let us set
As before, profit taxes enter only in the budget constraint, so we can ignore this constraint and the profit taxes are used to make sure that the budget constraint holds. We can substitute $y_t^S$, $y_t^F$, $w_t^F$, $w_t, c_t^S$, $c_t^F$ and $g_t^F$, such that the system on the aggregates $C_t, N_t$ can be written as

$$\frac{u_{1-N_t}}{u_{Ct}} \left[ \alpha + (1 - \alpha) \left( \frac{\theta w_t^S}{\theta - 1} s_t \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} = w_t^S (1 - \tau_d^t)$$

$$(C_t + G_t) \left( \alpha + (1 - \alpha) \left( \frac{(\theta - 1) s_t}{\theta w_t^S} \right)^\theta \right) = \left[ \alpha + (1 - \alpha) \left( \frac{(\theta - 1) s_t}{\theta w_t^S} \right)^{\theta-1} \right]^{\frac{\theta}{\theta-1}} s_t N_t$$

for all $(\sigma^t-1, \sigma_t)$, and

$$E_{t-1} \left[ \frac{u_{1-N_t}}{(1 - \tau_n^t)} \left( 1 - \tau_d^t \right) y_t^S \theta \frac{1}{\theta - 1} s_t \left( \frac{\theta - 1}{\theta} \right)^{\theta-1} \frac{-1}{\theta} \right] = 0.$$ 

The final set of equations necessary to determine policies and prices is the cash-in-advance constraint

$$\left[ \alpha P_t^{S(1-\theta)} + (1 - \alpha) P_t^{F(1-\theta)} \right]^{\frac{1}{1-\theta}} C_t = M_t$$

which can be written as

$$P_t^S \left[ \alpha + (1 - \alpha) \left( \frac{\theta w_t^S}{\theta - 1} s_t \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} C_t = \overline{M}_{t-1} \delta(\sigma^t-1, \sigma_t)$$

Thus, the three equations that determine the system become

$$\frac{u_{1-N_t}}{u_{Ct}} = \frac{W_{t}^{S}}{P_t^S} \left[ \alpha + (1 - \alpha) \left( \frac{\theta w_t^S}{\theta - 1} s_t \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} (1 - \tau_n^t)$$

$$C_t + G_t = \left[ \alpha + (1 - \alpha) \left( \frac{\theta W_t^S}{(\theta - 1) s_t P_t^S} \right)^{(1-\theta)} \right]^{\frac{\theta}{\theta-1}} s_t N_t$$

\footnote{As before the expected component of monetary policy $\mu(\sigma^t)$ is the instrument used to induce $R_{t+1} = 1$.}
\[ P_t^S \left[ \alpha + (1 - \alpha) \left( \frac{\theta W_t}{(\theta - 1) s_t P_t^s} \right)^{(1-\theta)} \right] \frac{1}{\theta} C_t = M_{t-1} \delta(\sigma^{t-1}, \sigma_t) \]

for all \((\sigma^{t-1}, \sigma_t)\), plus the restrictions

\[ E_{t-1}(\delta(\sigma^{t-1}, \sigma_t)) = 1 \]

and

\[ E_{t-1} \left[ \frac{u_1 - N_t}{(1 - \tau_t^p)} (1 - \tau_t^q) y_t^S \frac{\theta}{\theta - 1} \frac{1}{s_t} \left( \frac{(\theta - 1) s_t P_t^a}{\theta W_t} - 1 \right) \right] = 0 \]

Note, however, that the relative wages chosen in the optimal policy problem must satisfy this last condition, since it is a constraint on the set of feasible allocations. Thus, we have \(3N+1\) equations to determine \(\delta(\sigma^{t-1}, \sigma_t), W(\sigma^{t-1}, \sigma_t), \tau_t^p(\sigma^{t-1}, \sigma_t)\) for all \(\sigma_t\), and \(P_t^S(\sigma^{t-1})\). Thus, there is a unique solution.\(^{10}\)

Let \(z_t = \frac{\theta W_t}{(\theta - 1) s_t P_t^s} \). Note that we can think of the allocation being pinned down by the first two equations by means of the labor income tax rate, given a value for \(z\). Note also that we can think of the second equation as the effective production possibilities frontier, that depends on the exogenous shock and on the choice of \(z\). On the other hand, if the social welfare function is increasing on consumption and decreasing on labor, the optimal value for \(z\) is the one that maximizes the value for

\[ f(z_t) = \frac{\left[ \alpha + (1 - \alpha) z_t^{\theta - 1} \right] z_t^\phi}{(\alpha + (1 - \alpha) z_t^\phi)} \]

Then, given that value for \(z\), the labor income tax pins down the preferred point in the frontier and the third equation tells us, given \(z\) and the particular allocation pinned down, which is the quantity of money that implements the optimal value for \(z\).

It turns out, that the function \(f(z)\) satisfies the following properties

\(^{10}\)Recall that the solution is unique since we imposed \(R_{t+1} = 1, \tau_t = 0\). Obviously, the policy is not unique in \(R_{t+1}, \tau_t^c, \tau_t^p\).
\begin{align*}
  f(0) &= \alpha^{\frac{1}{\theta}} \\
  \lim_{z \to \infty} f(z) &= (1 - \alpha)^{\frac{1}{\theta}} \\
  f'(z) &= \frac{\alpha(1 - \alpha)\theta z^{\theta - 2}}{(\alpha + (1 - \alpha) z_t^{\theta})^2} (\alpha + (1 - \alpha) z_t^{\theta - 1})^{\frac{1}{\theta - 1}} (1 - z)
\end{align*}

Thus, the function \( f(z) \) is maximized when \( z = 1 \).

Thus, optimal monetary policy must be set so as to make \( z_t = 1 \), for all \( t \) and all states. Thus amounts to making the monetary policy respond to shocks so as to keep the price level equal to the expected value. Then, fiscal policy must be set as if all prices were flexible.

### 4 Conclusions

The purpose of this paper is to compare the set of implementable allocations in economies with different price setting behavior. The main contribution of our paper is to study the relationship between policies and allocations in the dynamic Ramsey tradition, so the optimal fiscal and monetary policies are analyzed in an integrated approach. However, we focus our analysis on feasible sets, so most of our results are independent of government objectives.

The economies that we compare share several features: monopolistic competition, no capital and money introduced via a cash-in-advance constraint, and we assume that a fraction of the firms must set prices one period in advance. We then consider two main questions related to optimal policies. First, we consider the effect that the price setting restrictions have on the set of implementable allocations. Then, we study the extent to which the price setting restrictions affect the optimal policies.

The first finding of our paper is that the set of feasible allocations is invariant to the price setting behavior of firms. Second, we show that optimal policies are also invariant to the degree of price stickiness and that two simple principles describe optimal policies: fiscal policies should be set as if all prices were flexible, while monetary policy should replicate the flexible prices allocation, as if all prices were sticky.
References


